New results on leaf-critical and leaf-stable graphs

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Joint work with Kenta Ozeki and Carol Zamfirescu

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Hamiltonicity, traceablity and some genaralizations

All graphs are undirected, simple, and connected (unless stated otherwise).

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- The *path-covering number* $\mu(G)$ of G is the minimum number of vertex-disjoint paths covering G.
- The *branch number* $\mathrm{s}(G)$ of G is the minimum number of branches (vertices of degree at least 3) of the spanning trees of G.

Leaf-critical and leaf-stable graphs

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- $\forall v \in V(G) : \mathrm{ml}(G - v) = \mathrm{ml}(G)$ or
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Definition

Suppose $\text{ml}(G) = \ell$.

- G is *ℓ-leaf-stable*, if $\forall v \in V(G)$: $\text{ml}(G - v) = \ell$.
- G is *ℓ-leaf-critical*, if $\forall v \in V(G)$: $\text{ml}(G - v) = \ell - 1$.

Existence, $\ell = 2$

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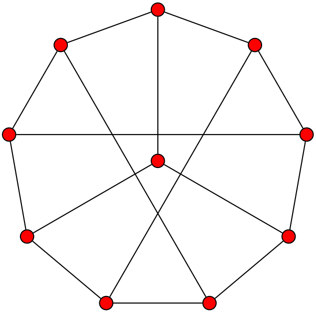

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- Flip-flop technique: 21, 23, 24, $\forall n \geq 26$ vertices [Chvátal, 1973]
- Examples if and only if $n = 10, 13, 15, 16$ and $\forall n \geq 18$ vertices [Herz-Duby-Vigue, 1967, Thomassen, 1974, Collier-Scmeichel 1978, Aldred-McKay-Wormald, 1997]

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- 105 vertices [Thomassen, 1976]
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- $\forall n \geq 42$ vertices [Jooyandeh, McKay, Östergård, Pettersson, C. Zamfirescu, 2014]

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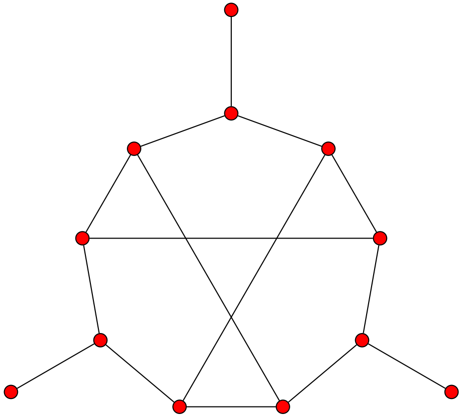
Gallai's problem

Smallest known counterexample to Gallai's question [Walther, T. Zamfirescu].



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- Smallest known hypotraceable by Thomassen (1974) on 34 vertices

Existence, arbitrary $\ell \geq 2$

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Theorem (W., 2017)

ℓ-leaf-critical and ℓ-leaf-stable graphs exist for every $\ell \geq 2$.

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- Construction based on J-cells
- Examples are 3-connected
- Even planar, cubic examples
- $\exists N$, s.t. $\forall n \geq N \; \exists$ example with n vertices

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A tree T is a *spider* if $s(T) \leq 1$. A spider is centred at the branch vertex (if there is no branch, then anywhere).

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Traceable and hypotraceable graphs are arachnoid.

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Question (Gargano et al., 2003)

Are there other arachnoid graphs?

Construction of other arachnoid graphs

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Theorem (W., 2017)

For every graph H there exists an arachnoid graph G that contains H as an induced subgraph, such that G is neither traceable, nor hypotraceable.

Fragments

Fragments

Definition

Let $G \neq K_n$, s.t. $\kappa(G) = k$, X be a cut of size k, and H be a component of $G - X$. Then $H + X$ is called a *k-fragment* of G, and X is the *vertices of attachment* of H.

Structure of leaf-critical and leaf-stable graphs

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- 2-leaf-critical: no, 3-leaf-critical: yes (Thomassen, 1974)

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- 2-leaf-stable: yes (W., 2017)

Leaf-critical and leaf-stable graphs of connectivity 2

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Theorem (Ozeki-C. Zamfirescu-W., 2018+)

Let $\ell \geq 3$, $F_1, \ldots, F_{\ell-1}$ be disjoint hypotraceable 2-fragments with vertices of attachment $\{x_i, y_i\}$ for $1 \leq i \leq \ell - 1$, respectively. Identifying y_i with x_{i+1} (mod $(\ell - 1)$), we obtain a graph G, which is ℓ-leaf-critical and of connectivity 2.

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Let z_i be the vertex obtained by identifying y_i with x_{i+1}.

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Theorem (Ozeki-C. Zamfirescu-W., 2018+)

Let G be the graph from the previous theorem. If each F_i has edge-connectivity 2, then $G + z_i z_j$ is $(\ell - 1)$-leaf-stable for $i \neq j$.

Leaf-stable graphs of connectivity 3

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Definition

Let H be a graph with a cubic vertex x s. t.

- H is non-hamiltonian.
- For every $v \in N(x)$ the graph $H - v$ is hamiltonian.
- For any edge e incident with x there is a hamiltonian x-path in H using e.

Then G is called *good* and x is called the special vertex of G.

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Definition

$G \cdot H_x$: for every $v \in V(G)$: take a copy H^v of H (and the copy x^v of x), and join the vertices of $G - v$ in $N_G(v)$ with the vertices of $H^v - x^v$ in $N_{H^v}(x^v)$ by an edge (using a bijection).

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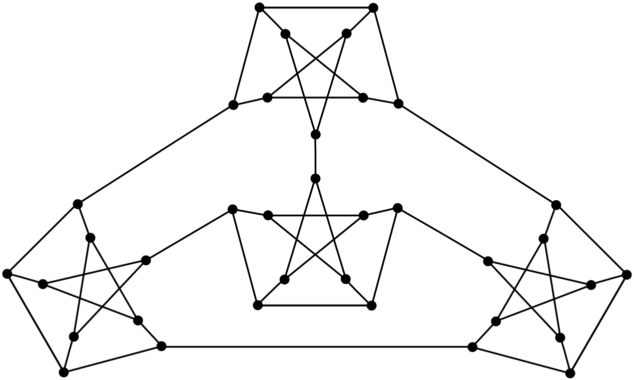

Leaf-stable graphs of connectivity 3

Theorem (Ozeki-C. Zamfirescu-W., 2018+)

Let G be a 2-edge-connected cubic graph and H good with special vertex x. Then $G \cdot H_x$ is $(|V(G)|/2 + 1)$-leaf-stable.

Smallest ℓ-leaf-stable and ℓ-leaf-critical graphs

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Definition

R^ℓ_κ (S^ℓ_κ): order of the smallest ℓ-leaf-critical (ℓ-leaf-stable) graph of connectivity κ. $\overline{R}^\ell_\kappa$ and $\overline{S}^\ell_\kappa$: planar case.

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- $\kappa \geq 4$:??? $\quad (\overline{R}^2_{\geq 4} = \infty$, by Tutte's thm)

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- $R^2_3 = 10$, $23 \leq \overline{R}^2_3 \leq 40$ (Goedgebeur-C. Zamfirescu, 2017; Jooyandeh et al., 2017)

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- $R^3_2 \leq 34$ (Thomassen, 1974), $R^3_3 \leq 40$ (Horton, 1973)

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- $R_3^2 = 10$, $\quad 23 \leq \overline{R}_3^2 \leq 40$ (Goedgebeur-C. Zamfirescu, 2017; Jooyandeh et al., 2017)
- $R_2^3 \leq 34$ (Thomassen, 1974), $\quad R_3^3 \leq 40$ (Horton, 1973)
- $\overline{R}_2^3 \leq 138$ (W., 2018), $\quad \overline{R}_3^3 \leq 200$ (Jooyandeh et al., 2017)

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- $\overline{R}_2^\ell \leq 69(\ell - 1)$ (Theorem 1)

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- $R^\ell_2 \leq 17(\ell - 1)$ (Theorem 1)
- $\overline{R}^\ell_2 \leq 69(\ell - 1)$ (Theorem 1)
- $R^\ell_3 \leq 16\ell - 8$ (W., 2017)

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- $\overline{R}_2^\ell \leq 69(\ell - 1)$ (Theorem 1)
- $R_3^\ell \leq 16\ell - 8$ (W., 2017)
- $\overline{R}_3^\ell \leq 76\ell - 38$ (W., 2017)

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- $S^2_2 = \overline{S}^2_2 = 12$ (Van Cleemput-C. Zamfirescu, 2017)

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- $S_2^2 = \overline{S}_2^2 = 12$ (Van Cleemput-C. Zamfirescu, 2017)
- $S_2^\ell \leq 17\ell, \quad \overline{S}_2^\ell \leq 69\ell$ (Theorem 2)

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- $S_2^\ell \leq 17\ell, \quad \overline{S}_2^\ell \leq 69\ell$ (Theorem 2)
- $S_3^\ell \leq \min\{18(\ell - 1), 16\ell\}$ (Theorem 3)

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Definition

R^ℓ_κ (S^ℓ_κ): order of the smallest ℓ-leaf-critical (ℓ-leaf-stable) graph of connectivity κ. $\overline{R}^\ell_\kappa$ and $\overline{S}^\ell_\kappa$: planar case.

- $S^2_2 = \overline{S}^2_2 = 12$ (Van Cleemput-C. Zamfirescu, 2017)
- $S^\ell_2 \leq 17\ell, \quad \overline{S}^\ell_2 \leq 69\ell$ (Theorem 2)
- $S^\ell_3 \leq \min\{18(\ell - 1), 16\ell\}$ (Theorem 3)
- $\overline{S}^\ell_3 \leq 46(\ell - 1)$ (Theorem 3)

Thank you.