Jochen Harant, Ilmenau, Germany

Let G be a graph on vertex set $V(G)$. If $S \subset V(G)$, then let $G - S$ be the graph obtained from G by removing S and $\omega(G - S)$ be the number of components of $G - S$. The *toughness* $t(G)$ *of* G (see [1]) is defined by

$$t(G) = \min_{S \subset V(G),\ \omega(G-S)>1} \frac{|S|}{\omega(G - S)}.$$

By a theorem of W.T. Tutte [4, 5, 6], a 4-connected planar graph is hamiltonian. Thus, a non-hamiltonian planar graph G contains $S \subset V(G)$ such that $|S| \leq 3$ and $\omega(G - S) \geq 2$ implying $t(G) \leq \frac{3}{2}$. It follows that a planar graph G is hamiltonian if $t(G) > \frac{3}{2}$.

Infinitely many non-hamiltonian planar graphs G with $t(G) = \frac{3}{2}$ are known (e.g. see [2]), however, no maximal planar graph is among them.

P. Owens [3] proved that, for arbitrary $\varepsilon > 0$, there is a non-hamiltonian maximal planar graph G with $t(G) > \frac{3}{2} - \varepsilon$.

Question: Does there exist a non-hamiltonian maximal planar graph G with $t(G) = \frac{3}{2}$?

References

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