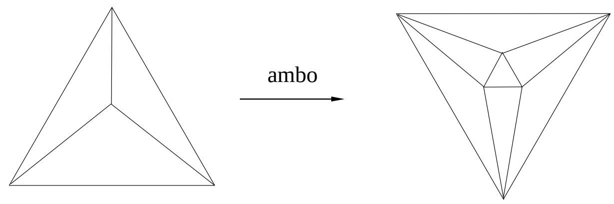
Which local operations can increase the symmetry?

Gunnar Brinkmann, Ghent University



Already Keppler and the ancient Greek knew local operations on polyhedra to construct larger polyhedra with the same symmetry group. The easiest of these operations are probably *dualizing* and *truncation*. On the wikipedia page for the *Conway polyhedron notation* many more example operations are given, but one must be careful: some of the operations given only preserve orientation preserving symmetries and can destroy orientation reversing symmetries. We are only interested in operations preserving all symmetries – no matter to which polyhedron they are applied. In this short text we talk about *local symmetry preserving operations* in a vague and intuitive way – a formal approach and a definition is given in [1].

Among all such operations there seems to be one that is special: *ambo* (also called *medial* or *rectification*). This operation also displayed in the figure above cuts off the corner points like *truncation* but in fact cuts deep: until the middle of the edges incident with the vertex cut off.

This operation **seems** to be the (more or less) only operation that does not only preserve all symmetries, but can in fact increase the symmetry group. If you apply this operation to a selfdual polyhedron, a symmetry coming from mapping the polyhedron to its dual is added to the symmetry group. In the figure above we apply ambo to the tetrahedron and get the more symmetrical octahedron.

In this context *“more or less”* means that one can construct other operations with this property – e.g. the product of ambo and truncation – but all known operations that can increase the symmetries of a polyhedron are of that kind: they can be written as a product involving ambo and it is ambo that really increases the symmetry and the other operations just preserve it.

Question: Is there a local symmetry preserving operation

- that can not be written as a product involving ambo,
- but there are polyhedra so that applied to those polyhedra the symmetries are not only preserved, but also new symmetries are introduced,

[1] Brinkmann, G.; Goetschalckx, P.; Schein, S. (2017). *Goldberg, Fuller, Caspar, Klug and Coxeter and a general approach to local symmetry-preserving operations*”. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences. 473 (2206): 20170267. arXiv:1705.02848